Gauge Symmetry Breaking with a Large Mass Hierarchy[†]

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We propose a higher dimensional scenario to solve the gauge hierarchy problem. In our formulation, a crucial observation is that a supersymmetric structure is hidden in the 4d spectrum of any gauge invariant theories with compact extra dimensions and that the gauge symmetry breaking is directly related to the supersymmetry breaking.

Recently, new possibilities of Grand Unified Theories (GUTs) in the context of higher dimensional field theories have extensively been explored to avoid common problems of four-dimensional GUTs. The GUT symmetry breaking scale is naturally given by the inverse length of the extra dimensions, but the origin of the weak scale is still a mystery.

In this paper, we propose a higher dimensional scenario to solve the hierarchy problem and explain how to get 4d masses much smaller than any scales of fundamental higher dimensional gauge theories without a fine tuning. Our mechanism to get hierarchically small masses is based on the idea that non-perturbative supersymmetry breaking can produce an exponentially small (supersymmetry breaking) mass scale such as

$$M \times e^{-\kappa} \lll M, \tag{1}$$

where M denotes a typical mass scale of the system and the exponential factor $e^{-\kappa}$ comes from a non-perturbative effect of supersymmetry breaking. Fortunately, a supersymmetric structure is already built in *any* gauge theories with compact extra dimensions [1, 2], and we can use it in our formulation. Then, the supersymmetry breaking immediately implies gauge symmetry breaking with the hierarchically small mass scale given by (1), which is a desired situation in constructing GUT models without the gauge hierarchy problem.

Let us begin to give a plausible argument for the statement that a supersymmetric structure is actually hidden in any gauge invariant theories with compact extra dimensions. To this end, let us consider a 4+1-dimensional gauge field $A_M(x,y) = (A_\mu(x,y), A_5(x,y))$. Expanding them into KK modes $(A_{\mu,n}(x), A_{5,n}(x))$, we find that the answers to the questions

- why there exists a 4d massless vector $A_{\mu,0}$?
- why massive modes $A_{5,n}$ are unphysical?
- why a massless mode $A_{5,0}$ (if exists) is physical?

can equally be explained in the languages of gauge symmetry and supersymmetry as follows:

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gauge symmetry point of view	supersymmetry point of view
The 4d gauge invariance ensures that the zero mode $A_{\mu,0}$ is massless.	Supersymmetry ensures that the vacuum state has zero energy.
All massive modes $A_{5,n}$ can be absorbed into the longitudinal modes of $A_{\mu,n}$ by gauge transformations.	The fact that the longitudinal modes of $A_{\mu,n}$ form superpartners with $A_{5,n}$ ensures that $A_{5,n}$ are gauged-away.
Gauge transformations cannot remove the zero mode $A_{5,0}$ because $A_5 \rightarrow A_5 + \partial_y \Lambda$.	Zero modes are special in SUSY because they do not, in general, form supermultiplets.

The above observations strongly suggest that the gauge symmetry is closely related to the supersymmetry. In fact, we can prove that an $N = 2$ supersymmetric quantum-mechanical structure exists in any gauge invariant theories with compact extra dimensions [2]. An explicit exercise will be given below (see also Ref. [1]).

In order to demonstrate how a hierarchically small mass arises in a higher dimensional gauge theory, let us consider a 4+1-dimensional gauge invariant theory on an interval $(0 \leq y \leq L)$ with the action

$$S = \int d^4x \int_0^L dy \Delta(y) \left\{ -\frac{1}{4} F_{MN}^a(x,y) F^{aMN}(x,y) \right\},$$ (2)

where $\Delta(y)$ is some weight function depending only on the 5th coordinate y. We here take $\Delta(y)$ to be of the form $\Delta(y) = \exp\left(-\frac{2\kappa}{L} y\right)$, as an illustrative example.

To obtain the 4d effective theory, we expand the gauge field $A_\mu(x,y)$ as $A_\mu(x,y) = \sum_n A_{\mu,n}(x) f_n(y)$, where

$$-\left[\frac{1}{\Delta(y)} \partial_y \Delta(y) \partial_y\right] f_n(y) = m_n^2 f_n(y),$$ (3)

while we expand $A_5(x,y)$ as $A_5(x,y) = \sum_n A_{5,n}(x) g_n(y)$, where

$$-\left[\partial_y \frac{1}{\Delta(y)} \partial_y \Delta(y)\right] g_n(y) = m_n^2 g_n(y).$$ (4)

Here, m_n corresponds to the mass of the 4d gauge boson $A_{\mu,n}$. It is easy to see that the eigenfunctions $f_n(y)$ and $g_n(y)$ are mutually related as

$$Q \begin{pmatrix} f_n \\ g_n \end{pmatrix} = m_n \begin{pmatrix} f_n \\ g_n \end{pmatrix}, \qquad Q = \begin{pmatrix} 0 & -\frac{1}{\Delta(y)} \partial_y \Delta(y) \\ \partial_y & 0 \end{pmatrix}.$$ (5)

The differential operator Q can be identified with a supercharge and in fact the above two differential equations, (3) and (4), can simply be combined into the form

$$H \begin{pmatrix} f_n \\ g_n \end{pmatrix} = m_n^2 \begin{pmatrix} f_n \\ g_n \end{pmatrix}, \qquad H = Q^2,$$ (6)

which is known as an $N = 2$ supersymmetric quantum mechanics [3].

This is not, however, the end of the story. Since the extra dimension has two boundaries at $y = 0$ and L, we have to specify some boundary conditions (BC's) there. The BC's should be compatible with gauge invariance. This requirement may be replaced by the statement that the BC's should be compatible with supersymmetry: for any wavefunction $\Psi(y)$ that obeys the required BC's, the state $Q\Psi(y)$ has to obey the same BC's, otherwise the supercharge Q is ill defined. It turns out that the above requirement severely restricts allowed BC's, and in fact only two types of BC's at each boundary are compatible with the supercharge Q [4]:

$$\text{Type N} : \partial_y f_n(y) = 0 \quad \text{and} \quad g_n(y) = 0, \tag{7}$$
$$\text{Type D} : f_n(y) = 0 \quad \text{and} \quad \partial_y \left(\Delta(y) g_n(y) \right) = 0, \tag{8}$$

at $y = 0, L$. It follows that there are 4 possible models with type (N,N), (D,D), (D,N) and (N,D), where the first (second) column denotes the BC's at $y = 0$ ($y = L$). In the first two models of type (N,N) and (D,D), there appears a massless mode and other KK modes acquire masses on the order of $\frac{1}{L}$. Thus, those models have nothing interesting. In the third model of type (D,N), there are no massless modes and all KK modes acquire masses on the order of $\frac{1}{L}$. This model is not a desired one. The last model of type (N,D) has no massless modes, but the lowest mode acquires a tiny mass

$$m \sim \frac{2\kappa}{L} e^{-\kappa}. \tag{9}$$

The remaining KK modes acquire masses on the order of $\frac{1}{L}$. Thus, the hierarchical structure in the spectrum is realized in this model. Since there is no massless mode, the 4d gauge invariance is broken, and the mass of the lightest gauge boson is given by (9). It is interesting to note that the supercharge Q is still well defined but there is no zero energy state with $Q|0\rangle = 0$ in this model. Therefore, we may say that supersymmetry is "spontaneously" broken. This observation may allow us to say that gauge symmetry is "spontaneously" broken in our mechanism, though the gauge symmetry breaking is caused by boundary effects.

In summary, we proposed a mechanism to break gauge symmetry with mass scales much smaller than any scales of underlying higher dimensional gauge theories without a fine tuning. A key observation in our formulation is that the problem of gauge symmetry breaking can be translated into that of supersymmetry breaking. It would be interesting to construct realistic GUT models without the hierarchy problem along the line discussed here.

References

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